

1. Find the derivative of $f(x) = 5^{x^2-3x}$.

Chain rule

$$f'(x) = 5^{x^2-3x} (\ln 5)(2x-3) \text{ or } (2x \ln 5 - 3 \ln 5) 5^{x^2-3x}$$

2. Find the derivative of $y = x^2 e^{-x}$.

Product rule

$$y' = 2x e^{-x} + x^2 (-1) e^{-x} = 2x e^{-x} - x^2 e^{-x} \text{ or } (2x - x^2) e^{-x}$$

$$\text{or } \frac{(2x - x^2)}{e^x}$$

3. Use implicit differentiation to find $\frac{dy}{dx}$.

$$3e^x + 2y - \ln \frac{x}{y} = 3$$

$$3e^x + 2y' - \frac{1}{x} + \frac{y'}{y} = 0 \quad 2y' + \frac{y'}{y} = \frac{1}{x} - 3e^x$$

Simplify first...

$$3e^x + 2y - (\ln x - \ln y) = 3$$

$$\left(2 + \frac{1}{y}\right) y' = \frac{1}{x} - 3e^x \quad y' = \frac{\frac{1}{x} - 3e^x}{2 + \frac{1}{y}}$$

$$3e^x + 2y - \ln x + \ln y = 3$$

Now simplify...yes, even if I don't tell you.

$$y' = \frac{\left(\frac{1}{x} - 3e^x\right) \cdot xy}{\left(2 + \frac{1}{y}\right) \cdot xy} = \frac{y - 3xye^x}{2xy + x}$$

4. Show if the function has an inverse and find the inverse. (if possible)

State the domain and *range* of each function.

$$g(x) = x^2 - 4 \text{ for } -\infty < x < \infty \text{ or } x \in \mathbb{R}.$$

$$g'(x) = 2x > 0 \text{ for } x > 0 \text{ therefore } g \text{ is strictly increasing and is one-to-one and has an inverse}$$

$$g'(x) = 2x < 0 \text{ for } x < 0 \text{ therefore } g \text{ is strictly decreasing and is one-to-one and has an inverse}$$

BUT, $g(x)$ is neither strictly increasing or decreasing throughout its entire domain so it has **no** inverse (not one to one)

Domain of g : $-\infty < x < \infty$ or $x \in \mathbb{R}$

Range of g : $[-4, \infty)$

5. Find the derivative of $y = (x+2)^{10}\sqrt{x-6}$ using logarithmic differentiation.

$$\ln y = \ln \left[(x+2)^{10}\sqrt{x-6} \right]$$

$$\ln y = \ln(x+2)^{10} + \ln(x-6)^{\frac{1}{2}}$$

$$\ln y = 10\ln(x+2) + \frac{1}{2}\ln(x-6)$$

$$\frac{y'}{y} = \frac{10}{x+2} + \frac{1}{2(x-6)}$$

$$y' = y \left[\frac{10}{x+2} + \frac{1}{2(x-6)} \right] = (x+2)^{10}\sqrt{x-6} \left[\frac{10}{x+2} + \frac{1}{2(x-6)} \right]$$

6. Evaluate $\int \frac{5^x}{5^x - 6} dx$ Let $u = 5^x - 6$
 $du = 5^x \ln 5 dx$
 $\frac{1}{\ln 5} du = 5^x dx$

$$\frac{1}{\ln 5} \int \frac{1}{u} du = \frac{1}{\ln 5} \ln |5^x - 6| + c = \frac{\ln |5^x - 6|}{\ln 5} + c$$

7. Find $f'(x)$ if $f(x) = \sin^{-1}(3x)$

$$f'(x) = \frac{1}{\sqrt{1-(3x)^2}} (3) = \frac{3}{\sqrt{1-9x^2}}$$

8. Evaluate $\int \frac{3}{x\sqrt{x^2-9}} dx = 3\left(\frac{1}{3}\right)\sec^{-1}\frac{x}{3} + c = \sec^{-1}\frac{x}{3} + c$ $a=3$

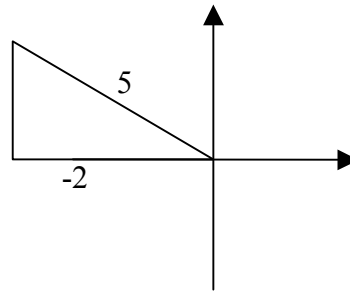
9. Evaluate $\int \frac{1}{x^2+2} dx = \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + c$ $a=\sqrt{2}$

10. Evaluate $\tan\left[\cos^{-1}\left(-\frac{2}{5}\right)\right]$ exactly.

$$\sqrt{5^2 - 2^2} = \sqrt{21}$$

$$\cos^{-1}\left(-\frac{2}{5}\right) = \theta$$

$$\cos \theta = -\frac{2}{5} = \frac{adj}{hyp} \text{ in Quadrant II}$$



$$\tan \theta = \frac{opp}{adj} = \frac{\sqrt{21}}{-2} = -\frac{\sqrt{21}}{2}$$

11. Evaluate $\int \frac{\ln x}{x} dx$

Let $u = \ln x$

$$du = \frac{1}{x} dx$$

$$\int \frac{\ln x}{x} dx = \int \ln x \cdot \frac{1}{x} dx = \int u du = \frac{1}{2} u^2 + c = \frac{1}{2} (\ln x)^2 + c$$

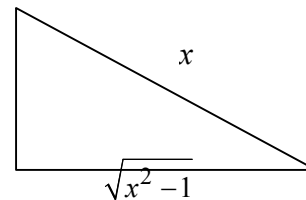
12. Rewrite the algebraic expression in x for $x > 0$.

$$\cos\left(\sin^{-1}\left(\frac{1}{x}\right)\right)$$

$$\sin^{-1}\left(\frac{1}{x}\right) = \theta$$

$$\sin \theta = \frac{1}{x} = \frac{opp}{hyp}$$

$$\cos \theta = \frac{adj}{hyp} = \frac{\sqrt{x^2 - 1}}{x}$$



13. List the domain and range of $y = \cos^{-1} x$.

Domain: $[-1, 1]$

Range: $[0, \pi]$

14. Evaluate $\int_1^2 e^x dx = e^x \Big|_1^2 = e^2 - e$