

14. Use this silly formula  $\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

Evaluate  $\int \sin 5x \cos 4x \, dx$  Here  $a = 5$  and  $b = 4$

$$\int \sin 5x \cos 4x \, du = -\frac{\cos(5-4)x}{2(5-4)} - \frac{\cos(5+4)x}{2(5+4)} + C = -\frac{\cos x}{2} - \frac{\cos 9x}{18} + C$$

**Extra Credit: No Stinking Calculators Either!!! PICK ONE. ONLY ONE!!!**

Evaluate  $\int \frac{1}{x(\sqrt{x} + \sqrt[4]{x})} \, dx$

$$\int \frac{4u^3 \, du}{u^4(\sqrt{u^4} + \sqrt[4]{u^4})} = 4 \int \frac{u^3 \, du}{u^4(u^2 + u)} = 4 \int \frac{du}{u(u^2 + u)} = 4 \int \frac{du}{u^2(u+1)}$$

Let

$$u = x^{\frac{1}{4}}$$

$$u^4 = x$$

$$4u^3 \, du = dx$$

Partial Fraction

$$4 \int \frac{1}{u^2(u+1)} \, du = 4 \int \left[ -\frac{1}{u} + \frac{1}{u^2} + \frac{1}{u+1} \right] \, du = 4 \left( -\ln|x^{\frac{1}{4}}| - \frac{1}{x^{\frac{1}{4}}} + \ln|x^{\frac{1}{4}} + 1| \right) + C = 4 \left( -x^{-\frac{1}{4}} + \ln \left| \frac{x^{\frac{1}{4}} + 1}{x^{\frac{1}{4}}} \right| \right) + C$$

Evaluate  $\int x\sqrt{5-3x} \, dx$

$$\int \frac{u-5}{-3} \sqrt{u} \left( -\frac{1}{3} \right) \, du = \frac{1}{9} \int (u-5)u^{\frac{1}{2}} \, du = \frac{1}{9} \int (u^{\frac{3}{2}} - 5u^{\frac{1}{2}}) \, du$$

Let

$$u = 5-3x$$

$$\frac{u-5}{-3} = x$$

$$-\frac{1}{3} \, du = dx$$

$$= \frac{1}{9} \cdot \frac{2}{5} u^{\frac{5}{2}} - \frac{5}{9} \cdot \frac{2}{3} u^{\frac{3}{2}} + C = \frac{2}{45} (5-3x)^{\frac{5}{2}} - \frac{10}{27} (5-3x)^{\frac{3}{2}} + C$$