What are Logarithms?? Why do we care? I do!!!

**Logarithms are exponents.**

**Definition:**

\[ x = b^y \iff y = \log_b x \]

*Most problems can be solved using the definition alone.*

**Example:**

\[ 5^x = 25 \text{ What is } x? \]

\[ 5^x = 5^2 \]

\[ x = 2 \]

What about \( 5^x = 24 \)?

*Not so easy*

**We could try to guess.** We know \( x \) is less than 2 and greater than 1. \( 1 < x < 2 \)

\[ x = \log_5 24 = \frac{\log 24}{\log 5} \]

\[ x \approx 1.380 \]

\[ \frac{1.380}{0.699} = 1.975 \]

**Change of Base**

\[ \log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b} \]

\[ \log_5 25 = \frac{\log 25}{\log 5} = \frac{\ln 25}{\ln 5} = 2 \]

*Base goes in the basement.*

**Let’s check that…**

\[ 5^{1.975} \approx 24.014 \]

Using more decimal places will give closer answer.

\[ 5^{1.974636} \approx 24.00000507 \]

*Since this log is irrational, we can only approximate.*

\[ \log_b 1 = 0 \quad \text{since} \quad 1 = b^0 \]

\[ \log_b b = 1 \quad \text{since} \quad b = b^1 \]

\[ \log_b b^x = x \quad \text{since} \quad b^x = b^x \]

**3 Basic Rules:**

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**NOTE:** \( \log 25 \neq \ln 25, \frac{\log 25}{\log 5} = \frac{\ln 25}{\ln 5} \) just like \( \frac{1}{2} = \frac{2}{4} \) (equivalent fractions)

Calculators are tricky. Older ones are backwards. Enter 25 then log. Modern ones: enter log 25.